Design of the Adrian Multi Radar Tracker in Phoenix for Enhanced Mode-S Surveillance

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Abstract

The topics out of sequence measurement and filter consistency for air traffic control (ATC) central tracker are discussed. The Kalman filter (KF) formulas of the Adrian-Multi-Radar-Tracker in the enhanced Mode-S version are presented. The current ideas on multi sensor fusion for multilateration and runway surveillance radar into AMRT finish the article.

Introduction

Civilian air transport wants to increase aircraft density - at least in the Frankfurt/Main airport area. This can hopefully be achieved with accurate fusion of existing surveillence radar data and use of additional ATC relevant data out of the aircraft flight management system (FMS) via Mode-S.
The AMRT is the tracker in the DFS arrival support system TWRTID and in the situation display system Phoenix. Development started in 1997. In 2002 Phoenix was evaluated by TEMPO for elementary Mode-S, in 2003 it is evaluated for enhanced Mode-S. Further development will integrate multilateration sensor data and runway surveillance radar (ASDE), into AMRT. TWRTID is in operation since 2001 at Frankfurt tower. Phoenix is in operation since 2002 at the towers Leipzig, Dresden, Erfurt and Zweibruecken.

Out of sequence (OOS) measurements

The time-in-storage of a long range (LR) radar measurement is approximately 1.2 seconds, the time-in-storage of a airport surveillance (ASR) radar measurement is approx. 0.6s. If an aircraft (AC) is near to a LR and far from an ASR, the more precise position measurement (plot) of the LR arrives later then the ASR plot at the central tracker.
The Kalman Filter (KF) formulas are valid if the difference of two plot times (t) is negative. Plots with t of less then -2 seconds are dropped. This is the only special processing of OOS measurements in the AMRT.
Implementation of further OOS special processing decreased the track quality. The author can not understand why the literature is full of "OOS solutions". Maybe some of them help in the area of filter inconsistency. The AMRT experience is that there is no correlation between having OOS measurements and filter inconsistency.

Statistical tests for filter consistency

In 2002 the AMRT failed one TEMPO evaluation run because the KF lost consistency. There were unexpected track terminations. The last few output values of the old track and the first few output values of the new track for the same AC had position errors above the acceptance limit. After implementation of chi-square (χ^2) distribution tests everything was fine.
A [x, y] measurement vector is tested against a χ^2 distribution with 2 degrees of freedom. A [z] measurement vector is tested against 1 degree of freedom.
The probability region is always 99.9%.

The single sided χ^2 distribution values for 99.9% confidence and 1 or 2 degrees of freedom are:

$qchisq(0.999, 1) = 10.828$

$qchisq(0.999, 2) = 13.816$

The normalized innovations squared (NIS) value ε_ν is calculated out of the measurement residual ν and the innovation covariance S. See Yaakov Bar-Shalom, Xiao-Rong Li; Estimation and Tracking: Principles, Techniques and Software; 1993 Artech House chapter 5.4.2.

$$\varepsilon_\nu = \nu^T \cdot S^{-1} \cdot \nu$$

A plot correlates to a track if the NIS is lower or equal the χ^2 limit, else the AMRT drops the plot.

Plot data

A plot has minimum two tracker relevant data values - range and azimuth. With enhanced Mode-S BDS code 5,0 and 6,0 the relevevant data values go up to 8. Because of different capabilities of the radar (with or without Mode-S) and of the AC FMS and transponder (without Mode-S, some BDS values, all BDS values) the tracker must use different KF formulas. Using only one KF formula and filling missing input values with defaults is not accurate.

Value	Range	Origin	Availability	Usage
range	0..200nmi	radar	always	position
azimuth	0..360°	radar	always	position
altitude	0..131000ft	AC	Mode-C	altitude
roll angle	-90°..+90°	AC	Mode-S BDS 5,0	no
true track angle	-180°..+180°	AC	Mode-S BDS 5,0	improve heading
ground speed	0..2046Knots	AC	Mode-S BDS 5,0	no
track angle rate	-16..+16°/s	AC	Mode-S BDS 5,0	improve turnrate
true airspeed	0..2046Knots	AC	Mode-S BDS 5,0	no
magnetic heading	-180°..+180°	AC	Mode-S BDS 6,0	improve heading
indicated airspeed	0..2046Knots	AC	Mode-S BDS 6,0	no
mach	0..4.096mach	AC	Mode-S BDS 6,0	no
baromet. altitude rate	-16384..+16352ft/min	AC	Mode-S BDS 6,0	improve altitude
inert. vertical velocity	-16384..+16352ft/min	AC	Mode-S BDS 6,0	improve altitude

The planar tracker for x, y position uses the Mode-S KF variation if one or more of the BDS values true track angle, track angle rate or magnetic heading are present. The altitude tracker for z position uses the Mode-S KF variation if one or more of the BDS values barometric altitude rate or inertial vertical velocity are present.
The speeds in BDS 5,0 and BDS 6,0 are not used. First tests with ground speed measurement as input into a position/velocity KF decreased track quality. Before new tests are done we will determine the standard deviation of ground speed information from real AC. Maybe the ground speed error was estimated to high.

Enhanced Mode-S Altitude Tracker

This KF has a two-dimensional measurement vector X_r with [z-position, z-speed]. The state vector X_s has two-dimensions, too. The notation is as used by Bar-Shalom, not as in the original paper of Kalman [R. E. Kalman; A New Approach to Linear Filtering and Prediction Problems; Transactions of the ASME-Journal of Basic Engineering; 82 (Series D) 1960; page 35-45].
The measurement noise matrix R depends on the Mode-C altitude error and the v_z error of the FMS. The U.S. FAA gives a standard deviation σ_z of 57ft for a Mode-C transponder with 100ft resolution. The process noise matrix q_c is assumed constant.

State prediction x(k+1|k) = F(k)*x(k|k)

$$X_p = \begin{pmatrix} z_p \\ vz_p \end{pmatrix} = F \cdot X_s = \begin{pmatrix} 1 & t_c \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} z_s \\ vz_s \end{pmatrix} = \begin{pmatrix} z_s + t_c \cdot vz_s \\ vz_s \end{pmatrix}$$

Measurement residual ν(k+1) = z(k+1) - H*x(k+1|k)

$$D = \begin{pmatrix} z_d \\ vz_d \end{pmatrix} = X_r - H \cdot X_p = \begin{pmatrix} z_r \\ vz_r \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} z_p \\ vz_p \end{pmatrix} = \begin{pmatrix} z_r - z_p \\ vz_r - vz_p \end{pmatrix}$$

State prediction covariance P(k+1|k) = F(k)*P(k|k)*F(k)' + G(k)*Q*G(k)'

$$P = \begin{pmatrix} p55 & p56 \\ p56 & p66 \end{pmatrix} = F \cdot P \cdot F^T + G \cdot Q_c \cdot G^T = \begin{pmatrix} 1 & t_c \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} p55 & p56 \\ p56 & p66 \end{pmatrix} \cdot \begin{pmatrix} 1 & t_c \\ 0 & 1 \end{pmatrix}^T + \begin{pmatrix} \frac{t_c^{\,2}}{2} & 0 \\ t_c & 0 \end{pmatrix} \cdot q_c \cdot \begin{pmatrix} \frac{t_c^{\,2}}{2} & 0 \\ t_c & 0 \end{pmatrix}^T$$

Measurement Error R

$$R = \begin{pmatrix} r55 & r56 \\ r56 & r66 \end{pmatrix} = \begin{pmatrix} \sigma_z^{\,2} & 0 \\ 0 & \sigma_{vz}^{\,2} \end{pmatrix}$$

Innovation covariance S(k+1) = H*P(k+1|k)*H' + R(k+1)

$$S = \begin{pmatrix} s55 & s56 \\ s56 & s66 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} p55 & p56 \\ p56 & p66 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}^T + \begin{pmatrix} r55 & r56 \\ r56 & r66 \end{pmatrix} = \begin{pmatrix} p55 + r55 & p56 + r56 \\ p56 + r56 & p66 + r66 \end{pmatrix}$$

Filter gain W(k+1) = P(k+1|k)*H*S(k+1)$^{-1}$

$$W = \begin{pmatrix} w_{55} & w_{56} \\ w_{65} & w_{66} \end{pmatrix} = P \cdot H \cdot S^{-1} = \begin{pmatrix} p55 & p56 \\ p56 & p66 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} s55 & s56 \\ s56 & s66 \end{pmatrix}^{-1}$$

Updated state estimate x(k+1|k+1) = x(k+1|k)+W(k+1)*ν(k+1)

$$X_s = \begin{pmatrix} z_s \\ vz_s \end{pmatrix} = X_p + W \cdot D = \begin{pmatrix} z_p \\ vz_p \end{pmatrix} + \begin{pmatrix} w_{55} & w_{56} \\ w_{65} & w_{66} \end{pmatrix} \cdot \begin{pmatrix} z_d \\ vz_d \end{pmatrix} = \begin{pmatrix} z_p + w_{55} \cdot z_d + w_{56} \cdot vz_d \\ vz_p + w_{65} \cdot z_d + w_{66} \cdot vz_d \end{pmatrix}$$

Updated state covariance P(k+1|k+1) = P(k+1|k) - W(k+1)*S(k+1)*W(k+1)'

$$P = \begin{pmatrix} p55 & p56 \\ p56 & p66 \end{pmatrix} = P - W \cdot S \cdot W^T = \begin{pmatrix} p55 & p56 \\ p56 & p66 \end{pmatrix} - \begin{pmatrix} w55 & w56 \\ w65 & w66 \end{pmatrix} \cdot \begin{pmatrix} s55 & s56 \\ s56 & s66 \end{pmatrix} \cdot \begin{pmatrix} w55 & w56 \\ w65 & w66 \end{pmatrix}^T$$

Note: The AMRT implementation language C does not have build-in matrix operations. The matrix formulas are expanded into elementary calculations. Only the upper triangle of the matrices P and S is actually calculated, the lower triangle is mirrored. P should therefore not tend to have negative eigenvalues due to rounding effects, the Joseph form covariance update is not needed.

Enhanced Mode-S Planar Tracker

This KF has a two-dimensional measurement vector X_r with [x-position, y-position]. The state vector X_s has four-dimensions with [x-position, x-speed, y-position, y-speed].

The turnrate ω is used as an input into the KF, but not as a state variable. This approach is used because only a minority of radars will be equipped with Mode-S and can deliver track angle rate, the source of turnrate. Remember: feeding more state variables with the same amount of information results in less accuracy per state variable.

To calculate the measurement error matrix R the errors are projected from sensor coordinate system to tracking coordinate system. See [Bar-Shalom, Li; Multitarget-Multisensor Tracking: Principles and Techniques; Yaakov Bar-Shalom; 1995] chapter 1.6.2. The measurement value range ρ has standard deviation error σ_ρ, the azimuth θ has error σ_θ. The measurement standard deviations depend on the radar (ASR or LR type, MSSR or Mode-S capability). The process noise matrix q is constant. For civilian AC q is typically 0.4gravo squared (15.4 m^2/s^4).

State prediction x(k+1|k) = F(k)*x(k|k)

$$c12 = \frac{\sin(\omega \cdot t)}{\omega} \quad c13 = \frac{1 - \cos(\omega \cdot t)}{\omega} \quad c22 = \cos(\omega \cdot t) \quad c24 = \sin(\omega \cdot t)$$

$$X_p = \begin{pmatrix} x_p \\ vx_p \\ y_p \\ vy_p \end{pmatrix} = F \cdot X_s = \begin{pmatrix} 1 & c12 & 0 & -c13 \\ 0 & c22 & 0 & -c24 \\ 0 & c13 & 1 & c12 \\ 0 & c24 & 0 & c22 \end{pmatrix} \cdot \begin{pmatrix} x_s \\ vx_s \\ y_s \\ vy_s \end{pmatrix} = \begin{pmatrix} x_s + c12 \cdot vx_s - c13 \cdot vy_s \\ c22 \cdot vx_s - c24 \cdot vy_s \\ c13 \cdot vx_s + y_s + c12 \cdot vy_s \\ c24 \cdot vx_s + c22 \cdot vy_s \end{pmatrix}$$

Measurement residual ν(k+1) = z(k+1) - H*x(k+1|k)

$$D = \begin{pmatrix} x_d \\ y_d \end{pmatrix} = X_r - H \cdot X_p = \begin{pmatrix} x_r \\ y_r \end{pmatrix} - \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} x_p \\ vx_p \\ y_p \\ vy_p \end{pmatrix} = \begin{pmatrix} x_r - x_p \\ y_r - y_p \end{pmatrix}$$

State prediction covariance P(k+1|k) = F(k)*P(k|k)*F(k)' + G(k)*Q*G(k)'

$$P = \begin{pmatrix} 1 & c12 & 0 & -c13 \\ 0 & c22 & 0 & -c24 \\ 0 & c13 & 1 & c12 \\ 0 & c24 & 0 & c22 \end{pmatrix} \cdot \begin{pmatrix} p11 & p12 & p13 & p14 \\ p12 & p22 & p23 & p24 \\ p13 & p23 & p33 & p34 \\ p14 & p24 & p34 & p44 \end{pmatrix} \cdot \begin{pmatrix} 1 & c12 & 0 & -c13 \\ 0 & c22 & 0 & -c24 \\ 0 & c13 & 1 & c12 \\ 0 & c24 & 0 & c22 \end{pmatrix}^T + \begin{pmatrix} \frac{t^2}{2} & 0 \\ t & 0 \\ 0 & \frac{t^2}{2} \\ 0 & t \end{pmatrix} \cdot \begin{pmatrix} q & 0 \\ 0 & q \end{pmatrix} \cdot \begin{pmatrix} \frac{t^2}{2} & 0 \\ t & 0 \\ 0 & \frac{t^2}{2} \\ 0 & t \end{pmatrix}^T$$

Measurement Error R

$$R = \begin{pmatrix} r11 & r12 \\ r12 & r22 \end{pmatrix} = \begin{bmatrix} {\sigma_\rho}^2 \cdot \sin(\theta)^2 + \rho^2 \cdot {\sigma_\theta}^2 \cdot \cos(\theta)^2 & \left({\sigma_\rho}^2 - \rho^2 \cdot {\sigma_\theta}^2\right) \cdot \sin\theta \cdot \cos\theta \\ \left({\sigma_\rho}^2 - \rho^2 \cdot {\sigma_\theta}^2\right) \cdot \sin\theta \cdot \cos\theta & {\sigma_\rho}^2 \cdot \cos(\theta)^2 + \rho^2 \cdot {\sigma_\theta}^2 \cdot \sin(\theta)^2 \end{bmatrix}$$

Innovation covariance S(k+1) = H*P(k+1|k)*H' + R(k+1)

$$S = \begin{pmatrix} s11 & s12 \\ s12 & s22 \end{pmatrix} = H \cdot P \cdot H^T + R = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} p11 & p12 & p13 & p14 \\ p12 & p22 & p23 & p24 \\ p13 & p23 & p33 & p34 \\ p14 & p24 & p34 & p44 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}^T + \begin{pmatrix} r11 & r12 \\ r12 & r22 \end{pmatrix}$$

Filter gain W(k+1) = P(k+1|k)*H*S(k+1)$^{-1}$

$$W = \begin{pmatrix} w11 & w12 \\ w21 & w22 \\ w31 & w32 \\ w41 & w42 \end{pmatrix} = P \cdot H^T \cdot S^{-1} = \begin{pmatrix} p11 & p12 & p13 & p14 \\ p12 & p22 & p23 & p24 \\ p13 & p23 & p33 & p34 \\ p14 & p24 & p34 & p44 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}^T \cdot \begin{pmatrix} s11 & s12 \\ s12 & s22 \end{pmatrix}^{-1}$$

Updated state estimate x(k+1|k+1) = x(k+1|k)+W(k+1)*ν(k+1)

$$X_s = \begin{pmatrix} x_s \\ vx_s \\ y_s \\ vy_s \end{pmatrix} = X_p + W \cdot D = \begin{pmatrix} x_p \\ vx_p \\ y_p \\ vy_p \end{pmatrix} + \begin{pmatrix} w11 & w12 \\ w21 & w22 \\ w31 & w32 \\ w41 & w42 \end{pmatrix} \cdot \begin{pmatrix} x_d \\ y_d \end{pmatrix} = \begin{pmatrix} x_p + w11 \cdot x_d + w12 \cdot y_d \\ vx_p + w21 \cdot x_d + w22 \cdot y_d \\ y_p + w31 \cdot x_d + w32 \cdot y_d \\ vy_p + w41 \cdot x_d + w42 \cdot y_d \end{pmatrix}$$

Updated state covariance P(k+1|k+1) = P(k+1|k) - W(k+1)*S(k+1)*W(k+1)'

$$P = \begin{pmatrix} p11 & p12 & p13 & p14 \\ p12 & p22 & p23 & p24 \\ p13 & p23 & p33 & p34 \\ p14 & p24 & p34 & p44 \end{pmatrix} = P - W \cdot S \cdot W^T$$

From multi radar to multi sensor

The extension of the KF formulas for multilateration sensors like the P3D from ERA (Czech republic) or NRN from DLR/TimeTech/Kayser-Threde (Germany) and runway surveillance radar like the ESR from FGAN/Atlas (Germany) is hopefully no big problem. The interface between sensor and central tracker neither. The ASTERIX plot or track formats and the internet protocol (IP) stack will likely be used.
Selecting a good approximation for measurement position and measurement errors projection from sensor coordinate system to tracking coordinate system will be the core of the job. Maintaining a common time scale between central tracker and sensors, too.
Last and most important is a quality check for the sensors. An error in the data of a precise sensor has more impact on the situation display that an error in a sensor with large error matrix. Typical engineering thinking is to blame the last change in a system for bad system performance. Applying this rule to a multi-sensor AMRT there will likely be a traditional tracker for the matured radar sensors only and a quality tracker that gets data from all sensors. If the data output of the quality tracker is near the data output of the traditional tracker the quality tracker output is used, else the traditional tracker output - with some warning to the system maintainer and/or to the system user. This two-tracker approach can only be used for targets that are seen by matured and new sensors. A van on the runway or a parking AC with Mode-S transponder switched off or not equipped is only seen by the runway surveillance radar.

The far view

The tools datalink and clever computer programs can hopefully support an ATC controller to higher performance, but only if these tools are always available and don't fail the human in a critical moment. An awful lot of testing is required for such a tool quality.
A human can change his behaviour after a short briefing. But how long does it take a computer program to change its behaviour? And up to this date, the computer will do the same mistake over and over again.

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Recorded Live Radar data playback

The playback shows two real aircraft doing 12°/s turns. Both aircraft start in the top left corner and perform a 540° left turn. The targets were seen by 8 radars. This results in an average measurement rate of 1.5 plots per second. The estimated positions were connected with straight lines to show the flight paths.